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Missile Autopilot Design using Adaptive CMAC Supervisory Controller

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***Abstract* — An adaptive CMAC-Supervisory (CMS) controller is proposed for aerodynamic missile pitch autopilot control. Missile motion is nonlinear and time-variant with unknown parameters. The controller is a combination of a supervisory controller and an adaptive CMAC (Cerebellar Model Articulation Controller). In the adaptive CMAC, a CMAC is used to approximate an ideal control law and a compensation controller to recover the residual of the approximation error. The supervisory controller is added to the adaptive CMAC to keep the system states within a predefined feasible set. The controller's stability verified with a Lyapunov function. Simulation results are carried out to confirm the efficiency of the proposed control.**

***Index Terms*— Missile autopilot, adaptive control, CMAC, supervisory control.**

I. INTRODUCTION

The principle of the pitch autopilot design is forcing the missile acceleration to track the acceleration command received from the guidance law. For that, several approaches have been developed over the years [1]-[5]. However, the dynamics of a tail controlled missile is non-minimum phase system. If we consider the angle of attack instead of the acceleration as output, this problem can be circumvented [6]-[8].

Recently, Artificial Neural Networks (ANNs) have become a hot research topic, due to their ability to solve nonlinear problems by learning [9]. In the aerospace engineering domain, ANNs have been applied successfully to flight control design, such as adaptive flight control [10]-[12], guidance law design [13], and missile autopilot design [14], [15]. Among many ANN architectures that have been proposed, the CMAC is the most popular neural model with distinguished features of being fast, simple to implement, solving nonlinear systems by the imposition of learning in offline mode [16]-[17], especially for modeling, system identification and control [18]. The advantages of using CMAC over other ANNs are well discussed in [19], [20]. The supervisory controllers have been proposed for stabilizing the system states around a predefined feasible set [21], [22].

In this paper, an adaptive CMS controller is proposed for designing a missile pitch autopilot which is aerodynamically controlled. Missile motion model is nonlinear and time-variant with unknown parameters, which could be due to errors in aerodynamic modelling. This controller is a combination between an adaptive CMAC and a supervisory controller [23]. The adaptive CMAC is presented to collaborate with the supervisory controller for stabilizing the system states around the predefined feasible set and satisfying the tracking performance. If the system states move away from the predefined feasible set, the supervisory controller starts working to push the states back, otherwise, it stays inactive. The controller's stability has been proved by using a Lyapunov function. A comparison between the feedback linearization controller and the proposed adaptive CMS controller is performed. The efficiency of the proposed controller is confirmed by simulation results.

This paper is organized as follows: the mathematical nonlinear model of the missile is introduced in section II. The design procedures of the proposed adaptive CMS controller are constructed in section III. Simulation results are provided to validate the efficiency of the proposed controller in section IV. Conclusions are drawn in Section V.

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II. NONLINEAR MISSILE MODEL

A. Missile model

Consider the first-order dynamics of actuator in the pitch plane, the nonlinear model of missile motion is given by [6]:

$$\begin{aligned}
\dot{\alpha} &= \left(\frac{\overline{Q}S}{mV}\right)\left[C_z(\alpha, M_m) + B_z\delta\right] + q \\
\dot{q} &= \left(\frac{\overline{Q}Sd}{I_{yy}}\right)\left[C_m(\alpha, M_m) + B_m\delta\right] \\
\dot{\delta} &= \omega_a\delta_c - \omega_a\delta
\end{aligned} \tag{1}$$

where α, q and δ are angle of attack, pitch rate and control fin deflection angle, respectively, and $\overline{Q}, S, m, V, d$ and I_{yy} are dynamic pressure, reference area, mass, velocity, reference length and pitching moment of inertia, respectively, and M_m represents Mach number. Also, δ_c and ω_a are control input and actuator bandwidth, respectively. The aerodynamic coefficients in Eq. (1) are described in terms of Mach number and angle of attack as:

$$\begin{aligned}
B_z &= b_1 M_m + b_2 \\
B_m &= b_3 M_m + b_4 \\
C_z(\alpha, M_m) &= \phi_{z1}(\alpha) + \phi_{z2}(\alpha) M_m \\
C_m(\alpha, M_m) &= \phi_{m1}(\alpha) + \phi_{m2}(\alpha) M_m \\
\phi_{z1}(\alpha) &= h_1\alpha^3 + h_2\alpha|\alpha| + h_3\alpha \\
\phi_{z2}(\alpha) &= h_4\alpha|\alpha| + h_5\alpha \\
\phi_{m1}(\alpha) &= h_6\alpha^3 + h_7\alpha|\alpha| + h_8\alpha \\
\phi_{m2}(\alpha) &= h_9\alpha|\alpha| + h_{10}\alpha
\end{aligned} \tag{2}$$

where b_i and h_i are the constant coefficients.

From Eqs. (1) and (2), we get

$$\begin{aligned}\dot{x}_1 &= f_1(x_1) + x_2 + g_1 x_3 \\ \dot{x}_2 &= f_2(x_1) + g_2 x_3 \\ \dot{x}_3 &= \omega_a u - \omega_a x_3\end{aligned} \tag{3}$$

where

$$x_1 = \alpha, \quad x_2 = q, \quad x_3 = \delta, \quad u = \delta_c$$
$$f_1(x_1) = C_1[\phi_{z1}(x_1) + \phi_{z2}(x_1) M_m]$$
$$f_2(x_1) = C_2[\phi_{m1}(x_1) + \phi_{m2}(x_1) M_m]$$
$$g_1 = C_1 B_z, \quad g_2 = C_2 B_m, \quad C_1 = \frac{\overline{Q}S}{mV}, \quad C_2 = \frac{\overline{Q}Sd}{I_{yy}} \tag{4}$$

B. Aerodynamics uncertainties modeling

Consider aerodynamics uncertainties and rewrite Eq. (3) as

$$\begin{aligned}\dot{x}_1 &= f_1(x_1) + \Delta f_1(x_1) + x_2 + [g_1 + \Delta g_1] x_3 \\ \dot{x}_2 &= f_2(x_1) + \Delta f_2(x_1) + [g_2 + \Delta g_2] x_3 \\ \dot{x}_3 &= \omega_a u - \omega_a x_3\end{aligned} \tag{5}$$

where

$$\begin{aligned}\Delta f_1(x_1) = {} & \Delta C_1 \phi_{z1}(x_1) + C_1 \Delta\phi_{z1}(x_1) + \Delta C_1 \Delta\phi_{z1}(x_1) + \Delta C_1 \\ & \cdot \phi_{z2}(x_1) M_m + C_1 \Delta\phi_{z2}(x_1) M_m + \Delta C_1 \Delta\phi_{z2}(x_1) M_m\end{aligned}$$
$$\begin{aligned}\Delta f_2(x_1) = {} & \Delta C_2 \phi_{m1}(x_1) + C_2 \Delta\phi_{m1}(x_1) + \Delta C_2 \Delta\phi_{m1}(x_1) + \Delta C_2 \\ & \cdot \phi_{m2}(x_1) M_m + C_2 \Delta\phi_{m2}(x_1) M_m + \Delta C_2 \Delta\phi_{m2}(x_1) M_m\end{aligned}$$
$$\Delta g_1 = \Delta C_1 B_z + C_1 \Delta B_z + \Delta C_1 \Delta B_z$$
$$\Delta g_2 = \Delta C_2 B_m + C_2 \Delta B_m + \Delta C_2 \Delta B_m$$
$$\Delta C_1 = \frac{\Delta\overline{Q}S}{mV}, \quad C_2 = \frac{\Delta\overline{Q}Sd}{I_{yy}}, \quad \Delta\overline{Q} = p_1 \overline{Q}$$
$$\Delta B_z = p_4 B_z, \quad \Delta B_m = p_5 B_m$$
$$|\Delta\phi_{zi}| \le p_2 |\phi_{zi}|, \quad |\Delta\phi_{mi}| \le p_2 |\phi_{mi}| \quad i = 1,2$$

and $p_i \ (i = 1, \ldots, 5)$ represent random constant perturbations.

From Eq. (5), it can be obtained that

$$\begin{cases}\ddot{x} = f(\mathrm{x}) + g(\mathrm{x}) u(t) + d(t) \\ y = x\end{cases} \tag{6}$$

where $x = x_1 = \alpha$

$$f(\mathrm{x}) = \frac{\partial f_1(x_1)}{\partial x_1}[f_1(x_1) + x_2 + g_1 x_3] + f_2(x_1) + g_2 x_3 - g_1 \omega_a x_3$$
$$g(\mathrm{x}) = g_1 \omega_a$$

$u(t) \in \Re$ and $y \in \Re$ are the control input and output, respectively, $d(t)$ is the aerodynamics uncertainties, and $\mathrm{x} = [x_1, x_2, x_3]^T \in \Re^3$ is a state vector of the system that is assumed to be available.

The objective is to design a missile pitch autopilot such that the output system y can track a given reference trajectory $y_d \in \Re$. We define a tracking error vector as

$$\boldsymbol{E} \equiv [e, \dot{e}]^T \tag{7}$$

where $e = y_d - y$ is the tracking error. If the parameters of the dynamic model and the aerodynamics uncertainties are available (i.e., the functions $f(\mathrm{x})$, $g(\mathrm{x})$ and $d(t)$ are known), then the so-called *Feedback Linearization* technique can solve the control problem [24]. In this case, the functions $f(\mathrm{x})$, $g(\mathrm{x})$ and $d(t)$ are used for construction of the ideal control law.

$$u^* = \frac{1}{g(\mathrm{x})}\left[-f(\mathrm{x}) - d(t) + \ddot{y}_d + \boldsymbol{K}^T \boldsymbol{E}\right] \tag{8}$$

where $K = [k_1, k_2]^T \in \Re^2$, in which $k_i (i = 1,2)$ are positive constants. Applying the control law (8) to system (6), the error dynamics is obtained.

$$\ddot{e} + k_1 \dot{e} + k_2 e = 0 \tag{9}$$

K is selected such that the real part of the solutions of $h(s) \equiv s^2 + k_1 s + k_2$ are strictly negative. This means that tracking of the reference trajectory is asymptotically achieved where $\lim e(t) \to 0$ when $t \to \infty$ for any starting initial conditions.

However, in practical applications, the exact knowledge of system $f(\mathrm{x})$, $g(\mathrm{x})$ and $d(t)$ is unavailable, which implies that the ideal control law (8) is unrealizable. Thus, in the following section, an adaptive CMS controller is proposed for designing a missile pitch autopilot.

III. Missile Autopilot Design based on Adaptive CMS Controller

Fig. 1 describes the configuration of the missile autopilot based on the adaptive CMS control system, which is composed of an adaptive CMAC and a supervisory controller. The control law takes the form.

$$u = u_A + u_S \tag{10}$$

where u_S is the output of the supervisory controller; $u_A = u_{CMAC} + u_C$ is the output of the adaptive CMAC, which consists of a CMAC u_{CMAC} and a compensation controller u_C. The supervisory controller can be conceived to push the states of the controlled system around a predefined feasible set; however, its performance is neglected. Therefore, the adaptive CMAC is presented to collaborate with the supervisory controller for stabilizing the system states around the predefined feasible set and satisfying the tracking performance.

A. Supervisory Controller

To control the divergence of states, it is necessary to design a supervisory controller. If the system states move away from the predefined feasible set, the supervisory controller starts working to push the states back, otherwise, it stays inactive. Only the adaptive CMAC will be used to imitate the ideal control law.

From Eqs. (6), (8) and (10), the error dynamics in the space-state form is obtained as follows:

$$\dot{E} = \Lambda E + G_m (u^* - u_A - u_S) \tag{11}$$

where

$$\Lambda = \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix} \text{ and } G_m = \begin{bmatrix} 0 \\ g(\mathrm{x}) \end{bmatrix}$$

Define the Lyapunov function as

$$V_S = \frac{1}{2} E^T P E \tag{12}$$

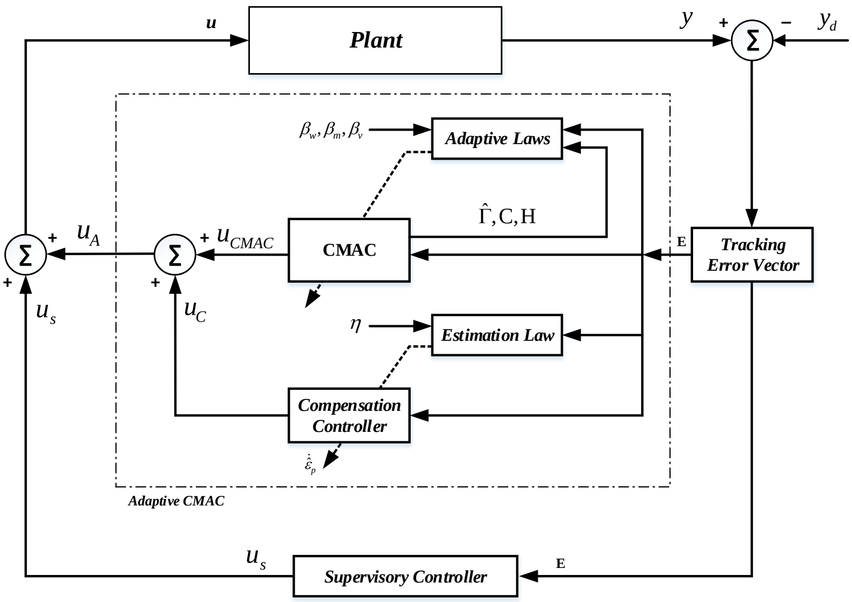

Fig. 1. Block diagram of the missile autopilot system based on the adaptive CMS controller.

where $P \in \Re^{n \times n}$ is a positive-definite symmetric matrix that satisfies the Lyapunov equation

$$\Lambda^T P + P\Lambda = -Q \tag{13}$$

and $Q \in \Re^{n \times n}$ is also a positive-definite symmetric matrix.

By using Eqs. (11), and (13) and taking the derivative of V_S with respect to time, we have

$$\begin{aligned}
\dot{V}_S &= \frac{1}{2}\dot{E}^T P E + \frac{1}{2}E^T P\dot{E} \\
&= \frac{1}{2}E^T \Lambda^T P E + \frac{1}{2}E^T P \Lambda E + E^T P G_m\left(u^* - u_A - u_S\right) \\
&= -\frac{1}{2}E^T Q E + E^T P G_m\left(u^* - u_A - u_S\right) \\
&\le -\frac{1}{2}E^T Q E + \left|E^T P G_m\right|\left(\left|u^*\right| + \left|u_A\right|\right) - E^T P G_m u_S
\end{aligned} \tag{14}$$

In order to formulate the supervisory control law u_S such that $\dot{V}_S \le 0$, it is necessary to know the bounds of the functions $f(\mathrm{x})$ and $g(\mathrm{x})$. Therefore, we make the following assumption.

Assumption: The bound functions $f^U(\mathrm{x}), g^U(\mathrm{x})$ and $g_L(\mathrm{x})$ are known such that $|f(\mathrm{x})| \le f^U(\mathrm{x})$ and $g_L(\mathrm{x}) \le g(\mathrm{x}) \le g^U(\mathrm{x})$ for all $\mathrm{x} \in U_C$, where $f^U(\mathrm{x}) < \infty$, $g^U(\mathrm{x}) < \infty$ and $g_L(\mathrm{x}) > 0$. Moreover, the aerodynamics uncertainties is bounded by $|d(t)| \le d^U$.

Based on the assumption and by observing Eqs. (8) and (14), the supervisory control law u_S is formulated as

$$\begin{aligned}
u_S &= I \cdot \operatorname{sgn}\left(E^T P G_m\right) \\
&\quad \cdot \left[\left|u_A\right| + \frac{1}{g_L(\mathrm{x})}\left(f^U(\mathrm{x}) + d^U + \left|\ddot{y}_d\right| + \left|K^T E\right|\right)\right]
\end{aligned} \tag{15}$$

where $\operatorname{sgn}(\cdot)$ is a sign function, and the operator index

$$I = \begin{cases} 1 & V_S \ge \bar{V} \\ 0 & V_S < \bar{V} \end{cases} \qquad \bar{V} \text{ is a positive constant.}$$

Substituting (8) and (15) into (14) and considering the case $I = 1$, yields

$$\begin{aligned}
\dot{V}_S &\le -\frac{1}{2}E^T Q E + \left|E^T P G_m\right|\left[\frac{1}{g(\mathrm{x})}\left(\left|f(\mathrm{x})\right| + \left|d(t)\right| + \left|\ddot{y}_d\right|\right.\right. \\
&\quad \left.\left. + \left|K^T E\right|\right) + \left|u_A\right| - \frac{1}{g_L(\mathrm{x})}\left(f^U(\mathrm{x}) + d^U + \left|\ddot{y}_d\right| + \left|K^T E\right|\right) - \left|u_A\right|\right]
\end{aligned}$$

$$\dot{V}_S \le -\frac{1}{2}E^T Q E \le 0 \tag{16}$$

Using the supervisory u_S controller presented in (15), when $V_S \ge \bar{V}$, the inequality $\dot{V}_S < 0$ can be obtained even for non-zero value of the tracking error vector E. From (16), the supervisory controller is capable of leading the tracking error to converge to zero.

However, due to the presence of sign function and the selection of the bounds $f^U(\mathrm{x}), g_L(\mathrm{x}), d^U$, an excessive and chattering control effort will be resulted. Moreover, the transient tracking performance may be not satisfied. Therefore, to overcome these phenomena, the adaptive CMAC will be formulated in the following subsection.

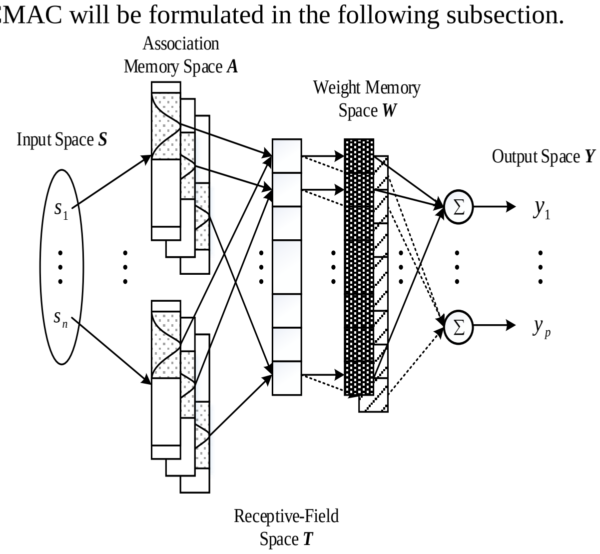

Fig. 2. Architecture of a CMAC

B. Implementation of CMAC

The architecture of CMAC is shown in Fig. 2, which includes an input space, an association memory space, a receptive-field space, a weight memory space and an output space [16], [25]. The signal propagation and the basic function in each space of CMAC are introduced as follows.

1) *Input Space S*

Consider the input space $S=[s_1,\ldots,s_n]\in\Re^n$. For a given control space, each input state variable s_i is quantified into discrete regions called *elements*. The number of elements, n_E, is designated as a resolution. In this design, the input state variables are

$$S=E=[e,\dot{e}]^T \tag{17}$$

2) *Association Memory Space A*

In this space, a block consists of several elements. The number of blocks, n_B, is generally larger than two. The operating principle of two-dimension CMAC is depicted in Fig. 3, with $n_E=9$ and $\rho=4$ (ρ is the number of elements in a full block), blocks A, B, and C divide the input state s_1, and blocks a, b, and c divide the input state s_2. New blocks will be obtained by shifting each variable an element. For example, blocks D, E, and F for s_1, and blocks d, e, and f for s_2 are obtained by such shifts.

Each block defines a *receptive-field basis function*, which can be represented as rectangular [16] or triangular or continuously bounded function (e.g., Gaussian [25], [26] or B-spline [27], [28]). Here, Gaussian function is formulated as the receptive-field basis function

$$\phi_{ik}(s_i)=\exp\left(-\frac{(s_i-m_{ik})^2}{v_{ik}^2}\right) \qquad \text{for } k=1,\ldots,n_B \tag{18}$$

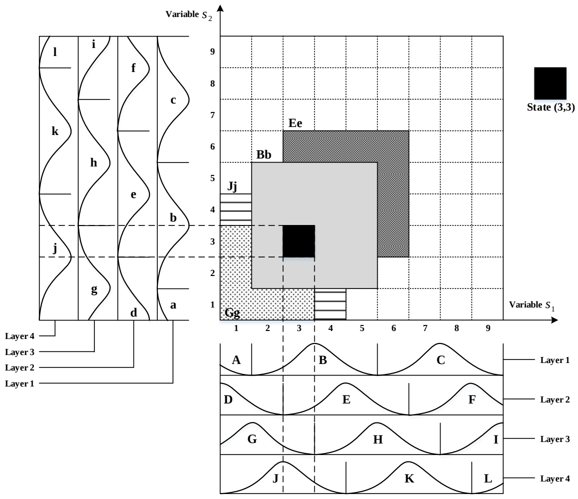

Fig. 3. CMAC in two-dimension with ρ = 4 and n_E = 9.

where $\phi_{ik}(s_i)$ represents the k^{th} block of the i^{th} input s_i with the mean m_{ik} and variance v_{ik}.

3) *Receptive-Field Space T*

Several blocks forms areas called *receptive-fields*. Each location of A is compatible with a receptive-field. Define the *multidimensional receptive-field function* as

$$b_k(S,\mathrm{m}_k,\mathrm{v}_k)=\prod_{i=1}^{n}\phi_{ik}(s_i)=\exp\left(\sum_{i=1}^{n}-\frac{(s_i-m_{ik})^2}{v_{ik}^2}\right),$$

$$\text{for } k=1,\ldots,n_R \tag{19}$$

n_R : Number of receptive-field.

where b_k is associated with the k^{th} receptive-field, $\mathrm{m}_k=[m_{1k},\ldots,m_{nk}]^T\in\Re^n$ and $\mathrm{v}_k=[v_{1k},\ldots,v_{nk}]^T\in\Re^n$.

The multidimensional receptive-field function can be represented as

$$\Gamma(S,\mathrm{m},\mathrm{v})=[b_1,\ldots,b_{n_R}]^T \tag{20}$$

where $\mathrm{m}=[\mathrm{m}_1^T,\ldots,\mathrm{m}_k^T,\ldots,\mathrm{m}_{n_R}^T]^T\in\Re^{n\times n_R}$

and $\mathrm{v}=[\mathrm{v}_1^T,\ldots,\mathrm{v}_k^T,\ldots,\mathrm{v}_{n_R}^T]^T\in\Re^{n\times n_R}$.

4) *Weight Memory Space W*

Each location of T is linked to a particular adjustable value in the weight memory space, can be represented as

$$\mathrm{w}=[\mathrm{w}_1,\ldots,\mathrm{w}_o,\ldots,\mathrm{w}_p]=\begin{bmatrix} w_{11} & \cdots & w_{1o} & \cdots & w_{1p}\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ w_{k1} & \cdots & w_{ko} & \cdots & w_{kp}\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ w_{n_R1} & \cdots & w_{n_Ro} & \cdots & w_{n_Rp}\end{bmatrix} \tag{21}$$

where $\mathrm{w}_o=[w_{1o},\ldots,w_{ko},\ldots,w_{n_Ro}]^T\in\Re^{n_R}$ and w_{ko} represents the connecting weight value of the o^{th} output associated with the k^{th} receptive-field.

The weight w_{ko} is initialized to zero and is automatically updated during online operation.

5) *Output Space Y*

The output of CMAC is the algebraic sum of the activated weights in the weight memory, and is represented as

$$y_o=\mathrm{w}_o^T\cdot\Gamma(S,\mathrm{m},\mathrm{v})=\sum_{k=1}^{n_R}w_{ko}\cdot b_k(S,\mathrm{m}_k,\mathrm{v}_k)$$

$$\text{for } o=1,\ldots,p \tag{22}$$

The outputs of the CMAC can be represented as

$$\mathbf{y}=[y_1,\ldots,y_o,\ldots,y_p]^T=\mathrm{w}^T\cdot\Gamma(S,\mathrm{m},\mathrm{v}) \tag{23}$$

C. Compensation Controller

Assume that there exists an optimal CMAC to approach to the ideal control law such that

$$u^*=u^*_{CMAC}(S,\mathrm{w}^*,\mathrm{m}^*,\mathrm{v}^*)\equiv\mathrm{w}^{*T}\cdot\Gamma^*+\varepsilon \tag{24}$$

where ε is the minimum approximation error; $\mathrm{w}^*,\mathrm{m}^*,\mathrm{v}^*$ and Γ^* are the optimal parameters of $\mathrm{w},\mathrm{m},\mathrm{v}$ and Γ, respectively.

Rewrite u_A as follows

$$u_A=u_{CMAC}(S,\hat{\mathrm{w}},\hat{\mathrm{m}},\hat{\mathrm{v}})+u_C\equiv\hat{\mathrm{w}}^T\cdot\hat{\Gamma}+u_C \tag{25}$$

where $\hat{\mathrm{w}},\hat{\mathrm{m}},\hat{\mathrm{v}}$ and $\hat{\Gamma}$ are the estimates of the optimal parameters of $\mathrm{w},\mathrm{m},\mathrm{v}$ and Γ. By subtracting (24) from (25), define an approximation error as

$$\begin{aligned}\tilde{u}=u^*-u_A&=\mathrm{w}^{*T}\cdot\Gamma^*+\varepsilon-\hat{\mathrm{w}}^T\cdot\hat{\Gamma}-u_C\\ &=\tilde{\mathrm{w}}^T\cdot\Gamma^*+\hat{\mathrm{w}}^T\cdot\tilde{\Gamma}+\varepsilon-u_C\end{aligned} \tag{26}$$

where $\tilde{\mathrm{w}}=\mathrm{w}^*-\hat{\mathrm{w}}$ and $\tilde{\Gamma}=\Gamma^*-\hat{\Gamma}$.

Based on Taylor theorem, and using the partial linear of the multidimensional receptive-field basis functions [29], [30], the expansion of $\tilde{\Gamma}$ becomes

$$\tilde{\Gamma}=\begin{bmatrix}\tilde{b}_1\\ \tilde{b}_2\\ \vdots\\ \tilde{b}_{n_R}\end{bmatrix}=\begin{bmatrix}\left(\frac{\partial b_1}{\partial \mathrm{m}}\right)^T\\ \left(\frac{\partial b_2}{\partial \mathrm{m}}\right)^T\\ \vdots\\ \left(\frac{\partial b_{n_R}}{\partial \mathrm{m}}\right)^T\end{bmatrix}_{|\mathrm{m}=\hat{\mathrm{m}}}(\mathrm{m}^*-\hat{\mathrm{m}})+\begin{bmatrix}\left(\frac{\partial b_1}{\partial \mathrm{v}}\right)^T\\ \left(\frac{\partial b_2}{\partial \mathrm{v}}\right)^T\\ \vdots\\ \left(\frac{\partial b_{n_R}}{\partial \mathrm{v}}\right)^T\end{bmatrix}_{|\mathrm{v}=\hat{\mathrm{v}}}(\mathrm{v}^*-\hat{\mathrm{v}})+O_t$$

$$\equiv C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}+O_t \tag{27}$$

where $\tilde{\mathrm{m}}=\mathrm{m}^*-\hat{\mathrm{m}}$, $\tilde{\mathrm{v}}=\mathrm{v}^*-\hat{\mathrm{v}}$ and $\tilde{b}_k=b_k^*-\hat{b}_k$.
b_k^* is the optimal parameter of b_k ; $\hat{b}_k$ is the estimated parameter of b_k^*;

$O_t\in\Re^{n_R}$ is a vector of higher-order terms;

$$C=\begin{bmatrix}\frac{\partial b_1}{\partial \mathrm{m}} & \frac{\partial b_2}{\partial \mathrm{m}} & \cdots & \frac{\partial b_{n_R}}{\partial \mathrm{m}}\end{bmatrix}\in\Re^{n\times n_R\times n_R}$$

$$H=\begin{bmatrix}\frac{\partial b_1}{\partial \mathrm{v}} & \frac{\partial b_2}{\partial \mathrm{v}} & \cdots & \frac{\partial b_{n_R}}{\partial \mathrm{v}}\end{bmatrix}\in\Re^{n\times n_R\times n_R}$$

where $\frac{\partial b_k}{\partial \mathrm{m}}$ and $\frac{\partial b_k}{\partial \mathrm{v}}$ are defined as

$$\left[\frac{\partial b_k}{\partial \mathrm{m}}\right]^T=\left[\underbrace{0,\ldots,0}_{(k-1)\times n},\frac{\partial b_k}{\partial m_{1k}},\ldots,\frac{\partial b_k}{\partial m_{nk}},\underbrace{0,\ldots,0}_{(n_R-k)\times n}\right] \tag{28}$$

$$\left[\frac{\partial b_k}{\partial \mathrm{v}}\right]^T=\left[\underbrace{0,\ldots,0}_{(k-1)\times n},\frac{\partial b_k}{\partial v_{1k}},\ldots,\frac{\partial b_k}{\partial v_{nk}},\underbrace{0,\ldots,0}_{(n_R-k)\times n}\right] \tag{29}$$

Rewrite (27) as

$$\Gamma^*=\hat{\Gamma}+C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}+O_t \tag{30}$$

Substituting (30) into (26), yields

$$\begin{aligned}\tilde{u}&=\tilde{\mathrm{w}}^T\left(\hat{\Gamma}+C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}+O_t\right)\\&\quad+\hat{\mathrm{w}}^T\left(C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}+O_t\right)+\varepsilon-u_C\\&=\tilde{\mathrm{w}}^T\hat{\Gamma}+\hat{\mathrm{w}}^T\left(C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}\right)+\tilde{\mathrm{w}}^T\left(C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}\right)\\&\quad+\mathrm{w}^{*T}O_t+\varepsilon-u_C\\&=\tilde{\mathrm{w}}^T\hat{\Gamma}+\hat{\mathrm{w}}^T\left(C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}\right)+D-u_C\end{aligned} \tag{31}$$

where $D=\tilde{\mathrm{w}}^T\left(C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}\right)+\mathrm{w}^{*T}O_t+\varepsilon$ represents the uncertain term and it is assumed to be bounded with a small positive constant ε_p (i.e., $|D|\le\varepsilon_p$). From (26) and (31), the error dynamics (11) can be rewritten in space state as

$$\begin{aligned}\dot{E}&=\Lambda E+G_m(\tilde{u}-u_S)\\&=\Lambda E+G_m\left(\tilde{\mathrm{w}}^T\hat{\Gamma}+\hat{\mathrm{w}}^T\left(C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}\right)-u_C+D-u_S\right)\end{aligned} \tag{32}$$

Theorem: Consider the nonlinear missile autopilot problem presented in (6). The adaptive CMS control system is designed as (10) where the supervisory controller is described in (15) and the adaptive CMAC is formulated in (25). Here, in the adaptive CMAC, the adaptive laws are chosen as (33)–(35) and the compensation controller as (36) with the estimation law given by (37), where β_w, β_m, β_v and η are strictly positive constants.

$$\dot{\hat{\mathrm{w}}}=\beta_w E^T PG_m\hat{\Gamma} \tag{33}$$

$$\dot{\hat{\mathrm{m}}}=\beta_m E^T PG_m C\hat{\mathrm{w}} \tag{34}$$

$$\dot{\hat{\mathrm{v}}}=\beta_v E^T PG_m H\hat{\mathrm{w}} \tag{35}$$

$$u_C=\hat{\varepsilon}_p\,\mathrm{sgn}\left(E^T PG_m\right) \tag{36}$$

$$\dot{\hat{\varepsilon}}_p=\eta\left|E^T PG_m\right| \tag{37}$$

Then the stability of the proposed controller is guaranteed.

Proof: Define a Lyapunov function as

$$\begin{aligned}&V\left(E,\tilde{\mathrm{w}},\tilde{\mathrm{m}},\tilde{\mathrm{v}},\tilde{\varepsilon}_p,t\right)\\&=\frac{1}{2}E^T PE+\frac{1}{2\beta_w}\tilde{\mathrm{w}}^T\tilde{\mathrm{w}}+\frac{1}{2\beta_m}\tilde{\mathrm{m}}^T\tilde{\mathrm{m}}+\frac{1}{2\beta_v}\tilde{\mathrm{v}}^T\tilde{\mathrm{v}}+\frac{1}{2\eta}\tilde{\varepsilon}_p^2\end{aligned} \tag{38}$$

where $\tilde{\varepsilon}_p=\varepsilon_p-\hat{\varepsilon}_p$ represents the estimation error of the uncertainty bound. Taking the derivative of V and using (13) and (32), it is concluded that

$$\begin{aligned}&\dot{V}\left(E,\tilde{\mathrm{w}},\tilde{\mathrm{m}},\tilde{\mathrm{v}},\tilde{\varepsilon}_p,t\right)\\&=\frac{1}{2}\dot{E}^T PE+\frac{1}{2}E^T P\dot{E}-\frac{1}{\beta_w}\tilde{\mathrm{w}}^T\dot{\hat{\mathrm{w}}}-\frac{1}{\beta_m}\tilde{\mathrm{m}}^T\dot{\hat{\mathrm{m}}}\\&\quad-\frac{1}{\beta_v}\tilde{\mathrm{v}}^T\dot{\hat{\mathrm{v}}}-\frac{1}{\eta}\tilde{\varepsilon}_p\dot{\hat{\varepsilon}}_p\\&=\frac{1}{2}E^T\left(\Lambda^T P+P\Lambda\right)E+E^T G_m P(\tilde{u}-u_S)-\frac{1}{\beta_w}\tilde{\mathrm{w}}^T\dot{\hat{\mathrm{w}}}\\&\quad-\frac{1}{\beta_m}\tilde{\mathrm{m}}^T\dot{\hat{\mathrm{m}}}-\frac{1}{\beta_v}\tilde{\mathrm{v}}^T\dot{\hat{\mathrm{v}}}-\frac{1}{\eta}\tilde{\varepsilon}_p\dot{\hat{\varepsilon}}_p\\&=-\frac{1}{2}E^T QE+E^T G_m P\left(\tilde{\mathrm{w}}^T\hat{\Gamma}+\hat{\mathrm{w}}^T\left(C^T\tilde{\mathrm{m}}+H^T\tilde{\mathrm{v}}\right)-u_C\right.\\&\quad\left.+D-u_S\right)-\frac{1}{\beta_w}\tilde{\mathrm{w}}^T\dot{\hat{\mathrm{w}}}-\frac{1}{\beta_m}\tilde{\mathrm{m}}^T\dot{\hat{\mathrm{m}}}-\frac{1}{\beta_v}\tilde{\mathrm{v}}^T\dot{\hat{\mathrm{v}}}-\frac{1}{\eta}\tilde{\varepsilon}_p\dot{\hat{\varepsilon}}_p\end{aligned} \tag{39}$$

From (15) and (33)–(37), (39) can be rewritten as

$$\begin{aligned}&\dot{V}\left(E,\tilde{\mathrm{w}},\tilde{\mathrm{m}},\tilde{\mathrm{v}},\tilde{\varepsilon}_p,t\right)\\&=-\frac{1}{2}E^T QE+E^T PG_m(D-u_C-u_S)-\frac{1}{\eta}\tilde{\varepsilon}_p\dot{\hat{\varepsilon}}_p\\&=-\frac{1}{2}E^T QE+E^T PG_m D-E^T PG_m u_C-E^T PG_m u_S\\&\qquad-\frac{1}{\eta}\left(\varepsilon_p-\hat{\varepsilon}_p\right)\dot{\hat{\varepsilon}}_p\\&\le-\frac{1}{2}E^T QE+E^T PG_m D-\hat{\varepsilon}_p\left|E^T PG_m\right|\\&\qquad-\left(\varepsilon_p-\hat{\varepsilon}_p\right)\left|E^T PG_m\right|\\&\le-\frac{1}{2}E^T QE-\left|E^T PG_m\right|\left(\varepsilon_p-|D|\right)\le 0\end{aligned} \tag{40}$$

Since $\dot{V}\left(E,\tilde{\mathrm{w}},\tilde{\mathrm{m}},\tilde{\mathrm{v}},\tilde{\varepsilon}_p,t\right)\le 0$ is a negative semi-definite function, $E,\tilde{\mathrm{w}},\tilde{\mathrm{m}},\tilde{\mathrm{v}}$ and $\tilde{\varepsilon}_p$ are all bounded. Consider the function $\Xi(t)=1/2\,E^T QE=-\dot{V}(t)$ and by integrating $\Xi(t)$ with respect to time

$$\int_o^t\Xi(\tau)\cdot d\tau\le V\left(E,\tilde{\mathrm{w}},\tilde{\mathrm{m}},\tilde{\mathrm{v}},\tilde{\varepsilon}_p,0\right)-V\left(E,\tilde{\mathrm{w}},\tilde{\mathrm{m}},\tilde{\mathrm{v}},\tilde{\varepsilon}_p,t\right) \tag{41}$$

Since $V\left(E,\tilde{\mathrm{w}},\tilde{\mathrm{m}},\tilde{\mathrm{v}},\tilde{\varepsilon}_p,0\right)$ is bounded, and $V\left(E,\tilde{\mathrm{w}},\tilde{\mathrm{m}},\tilde{\mathrm{v}},\tilde{\varepsilon}_p,t\right)$ is non-increasing function and bounded, it can concluded that:

$$\lim_{t\to\infty}\int_o^t\Xi(\tau)\cdot d\tau<\infty \tag{42}$$

Then, $\dot{\Xi}(t)$ is bounded, so based on Barbalat's Lemma [31], it can be noticed that $\lim_{t\to\infty} \Xi(t)=0$. So, $\lim_{t\to\infty} E(t)=0$. Consequently, the stability of the proposed controller is ensured.

IV. Numerical Simulations

To find out the efficiency of the proposed adaptive CMS controller for the missile pitch autopilot, simulations were carried out. The missile model used in simulation is a generic short-range surface-to-air missile and are its aerodynamic coefficients given in Table I. First-order actuator model $(\omega_a = 150\text{ rad/s})$ is considered.

The control objectives are as follows:

- Maintain stability over the operating range specified by $[\alpha(t), M_m(t)]$ such that

$$-10^\circ < \alpha(t) < 10^\circ \text{ and } 1.6 < M_m(t) < 2.6$$

- Track step command in α_C , with time constant ≈ 0.2 sec, less than 10 % overshoot and steady-state error no greater than 2 %.

Table I: Details of pitch axis missile model

$b_1 = 1.6238$	$h_5 = 4.185$ rad^{-1}
$b_2 = -6.7240$	$h_6 = 303.56$ rad^{-3}
$b_3 = 12.0393$	$h_7 = -246.3$ rad^{-2}
$b_4 = -48.2246$	$h_8 = -37.56$ rad^{-1}
$h_1 = -288.7$ rad^{-3}	$h_9 = 71.51$ rad^{-2}
$h_2 = 50.32$ rad^{-2}	$h_{10} = 10.01$ rad^{-1}
$h_3 = -23.89$ rad^{-1}	$C_1 = 0.2$
$h_4 = -13.53$ rad^{-2}	$C_2 = 50$

In all simulations, a 20% uncertainties existing in all random constant perturbation $p_i(i=1,\ldots,5)$ is taking into consideration.

A. Feedback Linearization Controller

For the purpose of comparison, the feedback linearization control law presented in (8) was simulated. The controller gains are chosen as $k_1 = 50$ and $k_2 = 225$.

Using the feedback linearization controller, the simulation results for a step command is depicted in Fig. 4. Fig. 4 (a) illustrates the AOA response and reference response for a step command. Also, Fig. 4 (b) - (d) depict the associated control effort, pitch rate and control fin deflection angle, respectively.

B. Adaptive CMS Controller

The adaptive CMS controller has been depicted in Fig. 1, which uses Gaussian function as receptive field basis functions. The input space was partitioned in a grid of size $1/2$, and receptive fields are selected to cover the input space $\{[-2,2],[-2,2]\}$ along with each of the input dimension. Therefore, the parameters are chosen as $v_{ik} = 2\sqrt{2}$ and $\text{m} = [-2.5, -1.5, -0.5, 0.5, 1.5, 2.5]$ for all *i* and *k*. The design parameters are set as follows:

$$Q = \begin{bmatrix} 400 & 15 \\ 15 & 1 \end{bmatrix}, \beta_w = 1.0, \beta_m = \beta_v = 0.75, \eta = 0.01, \bar{V} = 1$$

The adaptive CMS controller designed here needs to have the bounds f^U, g^U and g_L . In this system, $f^U = 104.3|x_3|$ and $g^U = g_L = 1$ are chosen.

Fig. 5 shows the simulation results under which the CMAC controller is designed alone, and Fig. 5(a) illustrates the AOA response and reference response for a step command. Also, Fig. 5(b) - (d) depict shows the associated control input, pitch rate and control fin deflection angle respectively. Although a good response is obtained, the chattering phenomena of the control efforts caused by the switching operation lead to the reduction of tracking accuracy.

Fig. 6 presents the simulation results of adaptive CMS controller, and Fig. 6(a) illustrates the AOA response and reference response for a step command. Also, Fig. 6(b)–(e) shows the associated control input, pitch rate, control fin deflection angle and the supervisory control, respectively. Note that Fig. 6(e) shows one activation period [0, 0.0022] sec. After 0.0022 sec, the supervisory is deactivated.

The comparison between three controllers is summarized in Table II, which shows that the adaptive CMS controller achieves the design requirement.

Table II: Controller's dynamic performances

Controller	Feedback Linearization	CMAC	CMAC-Supervisory
Settling time (s)	0.318	0.28	0.199
Overshoot (%)	4.8	-	0.17
Steady-state error	0.35 %	0.29 %	8.7×10^{-5} %

V. Conclusion

In this paper, an Adaptive CMAC Supervisory controller, including an adaptive CMAC and a supervisory controller, has been proposed to design a missile pitch autopilot for a nonlinear model which is aerodynamically controlled and contains unknown parameters and aerodynamic uncertainties. From the simulation results, the proposed control system achieves successfully the control objectives required. Future study will be applying this controller to other missile systems in order to further check its performance.

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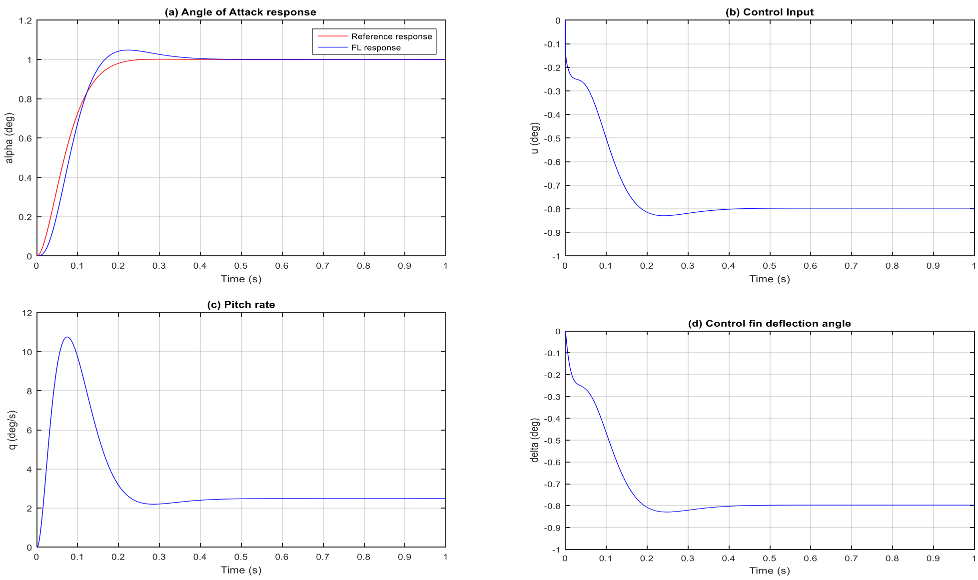
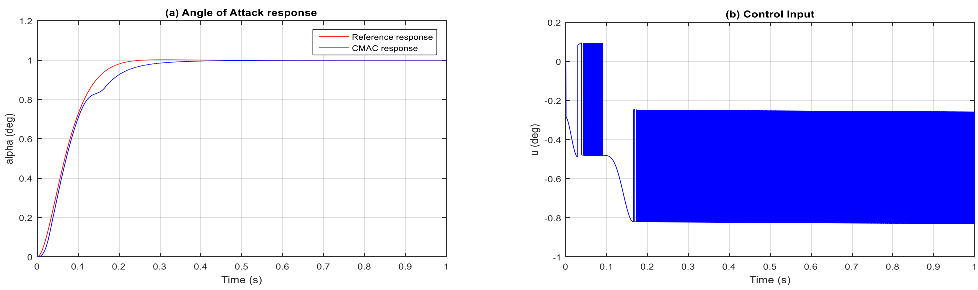


Fig.4. Simulation results for Feedback Linearization controller.



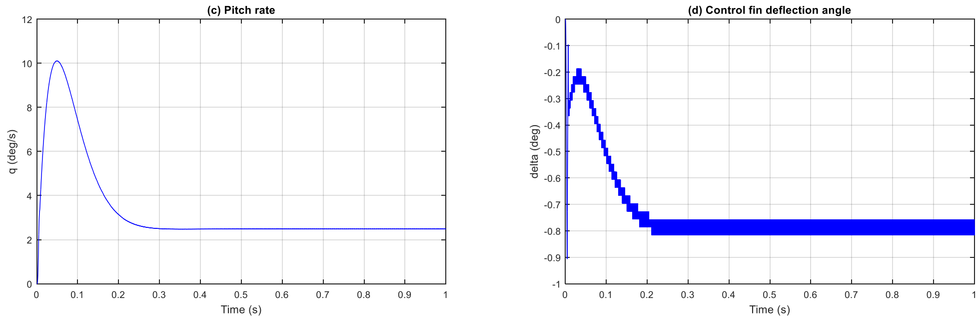


Fig.5. Simulation results for CMAC controller.

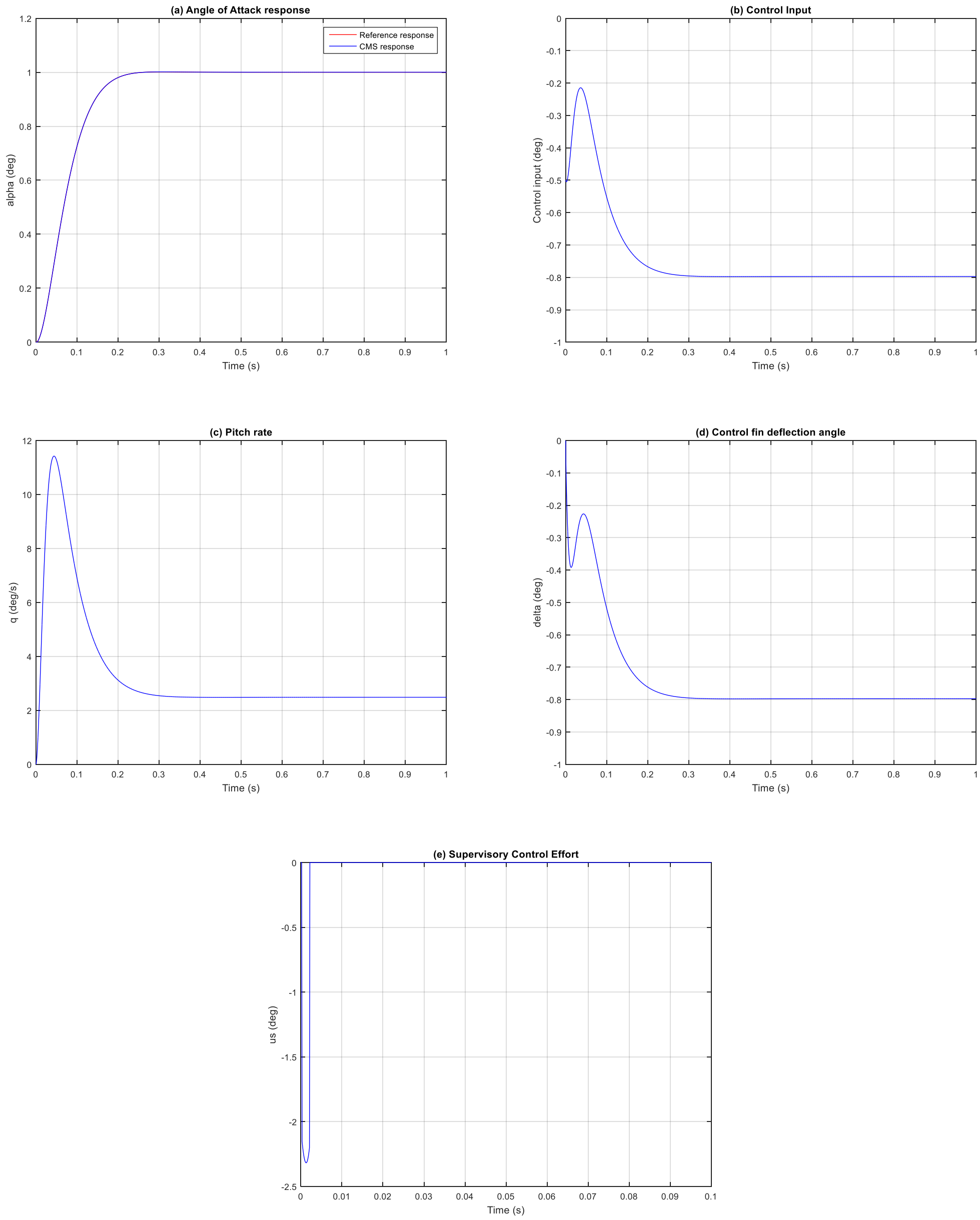


Fig.6. Simulation results for CMS controller.